

# First measurement of the double-inclusive $B/\bar{B}$ hadron energy distribution in $e^+e^-$ annihilations, and of angle-dependent moments of the $B$ and $\bar{B}$

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**Abstract.** We have made the first measurement of the double-inclusive  $B/\bar{B}$  energy distribution in  $e^+e^-$  annihilations, using a sample of 400,000 hadronic  $Z^0$ -decay events recorded in the SLD experiment at SLAC between 1996 and 1998. The small and stable SLC beam spot and the CCD-based vertex detector were used to reconstruct  $B/\bar{B}$ -decay vertices with high efficiency and purity, and to provide precise measurements of the kinematic quantities used to calculate the  $B$  energies in this novel technique. We measured the  $B/\bar{B}$  energies with good efficiency and resolution over the full kinematic range. We measured moments of the scaled energies of the  $B$  and  $\bar{B}$  hadrons vs. the opening angle between them. By comparing these results with perturbative QCD predictions we tested the ansatz of factorisation in heavy-quark production. A recent next-to-leading order calculation reproduces the data.

## 1 Introduction

The production of heavy hadrons ( $H$ ) in  $e^+e^-$  annihilation provides a laboratory for the study of heavy-quark ( $Q$ ) jet fragmentation. This is commonly characterized in terms of the observable  $x_H \equiv 2E_H/\sqrt{s}$ , where  $E_H$  is the energy of a  $B$  or  $D$  hadron containing a  $b$  or  $c$  quark, respectively, and  $\sqrt{s}$  is the c.m. energy. The distribution of  $x_H$ ,  $D(x_H)$ , is conventionally referred to as the heavy-quark ‘fragmentation function’<sup>1</sup>.

In recent publications we presented [1,2] the results of a new method for reconstructing  $B$ -hadron decays, and the  $B$  energy, inclusively, using only charged tracks, in the SLD experiment at SLAC. We have extended these studies and applied similar ‘topological’ vertexing techniques to tag events in which we reconstructed the energies of both leading  $B$  hadrons produced via  $e^+e^- \rightarrow b\bar{b} \rightarrow B\bar{B} + X$ . We measured the moments of the single-inclusive  $B$ -hadron scaled-energy distribution  $dN/dx_B$ :

$$D_i \equiv \int x_B^{i-1} \frac{1}{N_s} \frac{dN_s}{dx_B} dx_B \quad (1)$$

as well as the moments of the double-inclusive scaled-energy distribution:

$$D_{ij}(\phi) \equiv \int \int x_{B1}^{i-1} x_{B2}^{j-1} \frac{1}{N_d} \frac{d^3N_d}{dx_{B1} dx_{B2} d\cos\phi} dx_{B1} dx_{B2}, \quad (2)$$

<sup>1</sup> Unless stated otherwise, in these studies we do not distinguish between hadrons and antihadrons.

where  $x_{B1}$  and  $x_{B2}$  are the scaled energies of the two  $B$  hadrons and the label is arbitrary,  $\phi$  is the angle between their flight directions, and  $i$  and  $j$  are integers  $\geq 1$ . We formed the normalised moments:

$$G_{ij}(\phi) \equiv D_{ij}(\phi)/(D_i D_j) \quad (3)$$

Following the method proposed in [3] we used these quantities to test the ansatz of factorisation as applied to perturbative Quantum Chromodynamics (pQCD) calculations of  $e^+e^- \rightarrow b\bar{b}$  events. The  $G_{ij}$  can be derived at leading order (LO) in pQCD using the numerical results in [3]. In addition, next-to-leading order (NLO) predictions for  $G_{ij}$  have been calculated recently [4].

## 2 $B$ -hadron selection and energy measurement

This analysis is based on roughly 400,000 hadronic  $Z^0$  events produced in  $e^+e^-$  annihilations at a mean center-of-mass energy of  $\sqrt{s} = 91.28$  GeV at the SLAC Linear Collider (SLC), and recorded in the SLD Large Detector (SLD) between 1996 and 1998. A detailed description of the method can be found in [5].

The event sample for this analysis was selected using a ‘topological’ vertexing technique based on the detection and measurement of charged tracks, which is described in detail in [6,7,8]. We considered events in which we found decay vertices corresponding to both the leading  $B$  and

$\bar{B}$  hadrons. First, the Durham algorithm was applied to the selected hadronic events, with a  $y_c$  parameter value of 0.015, in order to define a jet structure in each event. We found that this algorithm and  $y_c$  value minimized the number of  $B$  (and  $D$ ) decay tracks assigned to the wrong jet. This is an important feature for our analysis because we used vertex-related variables derived only from charged tracks. Events containing 2, 3, or 4 jets were retained for further analysis.

In each selected event, the vertexing algorithm was applied to the set of tracks in each jet. Vertices consistent with photon conversions or  $K^0$  or  $\Lambda^0$  decays were discarded. Events were retained in which a vertex was found in exactly two jets. 35,137 events were selected, of which 89.4% were estimated to be of  $b\bar{b}$  origin. The efficiency for selecting true  $b\bar{b}$  events was estimated to 36.3%. Events were selected that contained at least one vertex with  $M_{Pt} > 2.0$  GeV/ $c^2$  and  $M_{Pt} \leq 2 \times M_{ch}$  [5]. The latter cut was found to reduce the contamination from fake vertices in light-quark events.

In order to improve the  $b\bar{b}$  purity of the sample, events were selected in which both vertices had a flight length,  $d_{vtx}$ , such that  $0.1 < d_{vtx} < 2.3$  cm; in which at least one vertex contained two ‘significant’ tracks, i.e. tracks with a normalised impact-parameter significance,  $d/\sigma_d$ , of at least 2 units; and in which the angle between the vertex flight vectors,  $\phi$ , satisfied  $\cos\phi < 0.99$ . The last cut was effective at removing events in which either a gluon had split into heavy quarks, or the jet-finder had artificially split a single heavy-quark jet into two jets.

The energy of each  $B$  hadron,  $E_B$ , can be expressed as the sum of the reconstructed-vertex energy,  $E_{ch}$ , and the energy of those particles not associated with the vertex,  $E_0$ . We can write

$$E_0^2 = M_0^2 + P_t^2 + P_{0l}^2 \quad (4)$$

The two unknowns,  $M_0$  and  $P_{0l}$ , must be found in order to obtain  $E_0$ . One kinematic constraint can be obtained by imposing the  $B$ -hadron mass on the vertex,  $M_B^2 = E_B^2 - P_B^2$ , where  $P_B = P_{chl} + P_{0l}$  is the total momentum of the  $B$  hadron. We derive the following inequality,

$$\sqrt{M_{ch}^2 + P_t^2} + \sqrt{M_0^2 + P_t^2} \leq M_B, \quad (5)$$

where equality holds in the limit where both  $P_{0l}$  and  $P_{chl}$  vanish in the  $B$  hadron rest frame. Equation (5) effectively sets an upper bound on  $M_0$ , and a lower bound is given by zero:

$$0 \leq M_0^2 \leq M_{0max}^2, \quad (6)$$

where

$$M_{0max}^2 = M_B^2 - 2M_B \sqrt{M_{ch}^2 + P_t^2} + M_{ch}^2. \quad (7)$$

Because  $M_0$  peaks near  $M_{0max}$ , [7] we set  $M_0^2 = M_{0max}^2$  if  $M_{0max}^2 \geq 0$ , and  $M_0^2 = 0$  if  $M_{0max}^2 < 0$ . We calculated  $P_{0l}$  and hence  $E_0$  (Equation (4)). We then reconstructed the  $B$  hadron energy,  $E_B^{rec} = E_0 + E_{ch}$ . Events were retained in which both reconstructed  $B$  energies satisfied  $E_B^{rec} <$

60 GeV. A final sample of 19,809 events was obtained with an estimated  $b\bar{b}$  selection efficiency of 21.7% and a background contribution of only 0.23%, which was almost entirely from  $Z^0 \rightarrow c\bar{c}$  events. We divided  $E_B^{rec}$  by the beam energy,  $E_{beam} = \sqrt{s}/2$ , to obtain the reconstructed scaled  $B$ -hadron energy,  $x_B^{rec} = E_B^{rec}/E_{beam}$ .

### 3 Angle-dependent $B\text{-}\bar{B}$ energy moments

In each event we quantified the correlations between the two  $B$  hadrons in terms of the angle dependent scaled-energy moments proposed in [3]. Using the raw measured scaled  $B$  energies we first evaluated the moments (1) from the raw measured distribution:

$$D_i^{rec} = \frac{1}{2N} \Sigma (x_B^{rec})^{i-1} \quad (8)$$

where the sum is over the set of reconstructed  $B$  hadrons and  $N$  is the number of events in the sample. Similarly, we evaluated in each  $\cos\phi$  bin the double moments (2):

$$D_{ij}^{rec}(\phi) = \frac{1}{N} \Sigma_{N(\phi)} (x_{B1}^{rec})^{i-1} (x_{B2}^{rec})^{j-1} \quad (9)$$

where the sum extends over the set of events in each  $\cos\phi$  bin. The normalised moments  $G_{ij}$  (3) were evaluated:

$$G_{ij}^{rec}(\phi) = \frac{D_{ij}^{rec}(\phi)}{D_i^{rec} D_j^{rec}} \quad (10)$$

The first six moments,  $i = 1, 2, 3$  and  $j = 1, \dots, i$  are shown in Fig. 1. The bin centers were defined by taking the average value of  $\cos\phi$  within each bin.

Also shown in Fig. 1 is a comparison with the simulated normalised moments; the simulation reproduces the data. Given that we showed previously [1, 2] that the Peterson function implemented in our simulation does not provide a good description of the  $b$ -quark fragmentation function, this agreement may naively appear to be surprising. However, if, as proposed in [3], the non-perturbative contributions to the normalised quantity  $G_{ij}$  cancel, the agreement should be excellent, as observed.

We used our simulated event sample to correct for the effects of the detector acceptance, the efficiency of the technique for reconstructing  $B$ -hadron decays, the energy resolution, and bin migrations caused by the finite angular resolution. We defined a binwise correction factor:

$$F_{ij}^{MC}(\phi) \equiv \frac{G_{ij}^{gen}(\phi)|_{MC}}{G_{ij}^{rec}(\phi)|_{MC}} \quad (11)$$

where  $G_{ij}^{gen}(\phi)|_{MC}$  is the moment calculated using generated true  $e^+e^- \rightarrow B\bar{B} + X$  events, and  $G_{ij}^{rec}(\phi)|_{MC}$  is the corresponding moment calculated after simulation of the detector and application of the same analysis as applied to the data. For  $\cos\phi \sim -1$  the value of  $F_{ij}$  is close to unity. As  $\cos\phi$  increases towards 1,  $F_{ij}$  rises monotonically to approximately 8 ( $F_{11}$ ) or 2 ( $F_{33}$ ). The increase with  $\cos\phi$

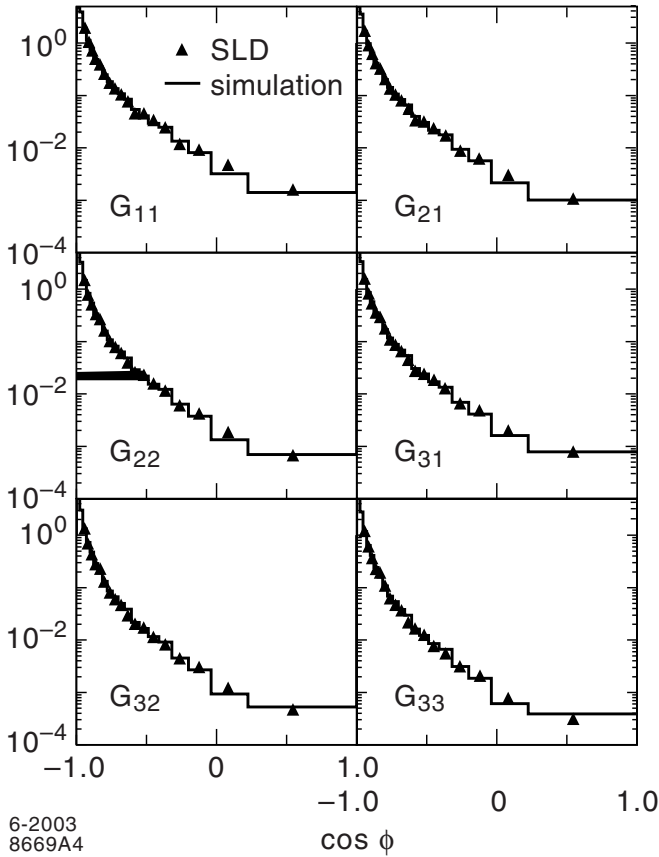


Fig. 1.

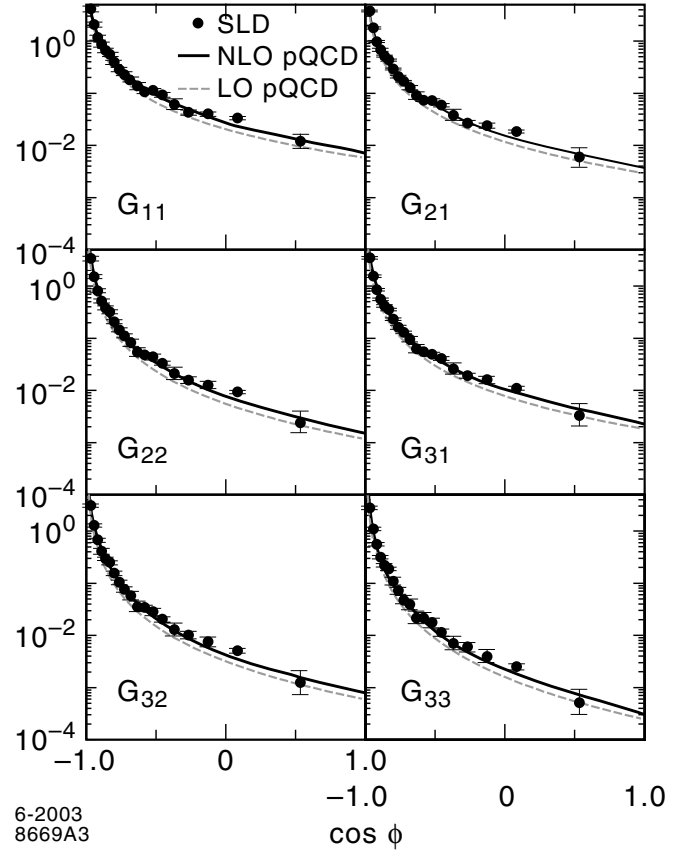


Fig. 2.

reflects our decreasing efficiency to select  $B\bar{B}$  events as the angle between the two  $B$ -decay vertices becomes smaller and the  $B$  energies decrease. For a given  $\cos\phi$  bin,  $F_{ij}$  decreases as  $i$  and  $j$  increase due to the effective weighting of this efficiency by  $x_B^{i-1}x_B^{j-1}$ .

We derived the true normalised moments:

$$G_{ij}(\phi) = F_{ij}^{MC}(\phi) G_{ij}^{rec}(\phi) \quad (12)$$

which are shown in Fig. 2. The statistical and systematic errors, including the uncertainties associated with this correction procedure are discussed in [5].

## 4 Comparison with perturbative QCD calculations

The fully-corrected data [8] were compared (Fig. 2) with a recent calculation [4] of the normalised moments complete at NLO in pQCD. Both the LO and full NLO calculations are shown in Fig. 2; the calculations assume an  $\alpha_s(M_Z^2)$  value of 0.120 and a pole  $b$ -quark mass value of 5.0 GeV/ $c^2$ . It can be seen that the difference between the two is relatively small, and that the LO calculation lies systematically slightly below the NLO calculation. The small size of the NLO relative to the LO contributions is an indication that the normalised moments are perturbatively robust observables. For each moment shown, the

LO calculation undershoots the data. The NLO calculation reproduces the data across the full range of  $\cos\phi$ .

This comparison does not rely on any convolution of the pQCD calculations with models of the non-perturbative hadronisation process. Hence the excellent agreement between the pQCD calculations and the data verifies the ansatz of factorization between the perturbative and non-perturbative phases that forms the basis for the pQCD calculation of heavy-hadron properties.

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